

Hypersonic Laminar Separation on a Cone with Slip Flow

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This is an experimental investigation of hypersonic, axisymmetric, laminar separation of a thick, slip-flow viscous layer on a cone that is isothermal and slightly cooled. Minutely blunted cones with half-angles of 5° and attached compression flares from 20° to 90° were used. The freestream had a Mach number of about 14.5 at the model tip and was slightly divergent. Experimental observations included surface-pressure measurements and Pitot surveys of attached and separated flows. These measurements are converted into approximate velocity profiles, Mach number profiles, displacement thicknesses, and shock shapes. The flows just upstream of separation interaction were influenced by 1) tip bluntness as far as 1000 tip diameters downstream of the tip, 2) weak interaction, and 3) the slight divergence of the freestream. Nevertheless, surface pressures and approximate displacement thicknesses for the separation-interaction regions on the cones are correlated by hypersonic, free-interaction, similarity variables that are obtained from a "Chapman analysis."

Nomenclature

C_f	= local skin-friction coefficient, τ_w/q_δ
C_{p0}	= $(p_w - p_0)/q_0$
C_∞	= Chapman-Rubens constant, $\mu_w T_\infty/\mu_\infty T_w$
d	= smallest outside dimension of probe tip, Fig. 3
L_∞	= idealized-source distance $(\gamma - 1)M_\infty/(\partial M/\partial x)_\infty$
l_i	= a characteristic length for separation interaction
l_{tr}	= length from ξ_0 to transition point
M	= Mach number
P	= pressure when freestream is uniform
$\bar{P}$	= pressure variable, Eq. (7)
p	= pressure when freestream is nonuniform
p_a	= local static pressure on tunnel centerline; nonuniform freestream; @ $x = 0$, $p_a \equiv p_\infty = P_\infty$
q	= dynamic pressure
Re	= unit Reynolds number
Re_{pr}	= Reynolds number at start of reattachment pressure rise, $\rho_\delta U_\delta(S - \xi_0)/\mu_\delta$
Re_{tr}	= transition Reynolds number based on plateau flow conditions, $\rho_\delta U_\delta l_{tr}/\mu_\delta$
Re_ξ	= $\rho_\delta U_\delta \xi/\mu_\delta$
S	= length of cone surface, Fig. 1
T	= temperature, °R
T_{aw}	= adiabatic wall temperature, °R
t	= tip bluntness, approximately diameter of hemisphere, in.
U	= velocity
$\bar{X}$	= distance variable, Eqs. (6) and (8)
x, y	= distance from tip along body axis of symmetry and normal to x axis, respectively, in., Fig. 1
Y	= distance from wall in y direction, in.
$\bar{Y}$	= similarity variable, Y/δ_0^*
α_b	= angle made by tangent to equivalent body for inviscid flow and x axis
γ	= ratio of specific heats
δ	= viscous-layer thickness, in.
δ^*	= axisymmetric displacement thickness, in.
$\bar{\delta}^*$	= displacement-thickness variable, δ^*/δ_0^*
Θ	= $(d\bar{\delta}^*/dx - [d\bar{\delta}^*/dx]_0)l_i/\delta_0^*$
θ_c	= cone half-angle, deg
λ	= Maxwellian mean free path, in.
μ	= viscosity

ξ	= distance from cone tip along body surface, in.
ρ	= density
τ_w	= local skin friction at wall
$\bar{\tau}$	= skin-friction variable, τ_w/τ_{w0}
ϕ	= flare angle, Fig. 1

Subscripts

o	= conditions at edge of viscous layer or other specified conditions at beginning of separation interaction
t	= total
w	= wall
δ	= conditions at edge of viscous layer
∞	= freestream conditions at model tip or related uniform freestream conditions

Introduction

THE purpose of this investigation is to experimentally examine a case of hypersonic, axisymmetric, laminar separation of a viscous layer that is initially about as thick as the body radius, but only slightly rarefied.

Prior investigations have been made of laminar, axisymmetric, compression separation at supersonic and hypersonic Mach numbers. However, this subject has not been as extensively studied as the corresponding two-dimensional problem. The following axisymmetric cases have received attention: 1) cylinder flares with various noses,¹⁻⁷ 2) spiked bodies,⁸⁻¹² and 3) cone flares.¹³⁻¹⁶ Several of these investigations^{7,10-12} involved separations of thick, hypersonic, rarefied, viscous layers. However, these investigations did not include detailed experimental observations of the separation-interaction process. The separation-interaction region is defined as that region which extends from the beginning of a separation-induced surface-pressure rise to the start of a reattachment pressure rise or, when present, to the beginning of a surface-pressure plateau. The investigation reported here concentrates on the flow in this region.

Test Conditions, Apparatus, and Procedures†

Models

The models (Figs. 1 and 2) consisted of minutely blunted right-circular cones of 5° half-angle and attached flares from 20° to 90° relative to the axis of symmetry. Surface temperatures were measured 1.25 in. from the cone tip, at the flare-cylinder juncture, and on the model base. The silver and copper surfaces (Fig. 2) were isothermal and slightly

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† Most of the methods, uncertainties, and corrections are discussed in greater detail in Ref. 17. The author estimates that there is about a 95% chance that errors will fall within the uncertainty intervals given in this section.

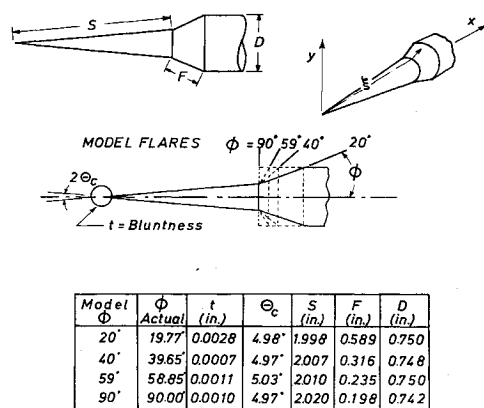


Fig. 1 Model geometry.

cooled, since $T_w = 1460 \pm 10^\circ\text{R}$ and $T_w/T_{aw} \approx 0.82$. The temperatures of the stainless-steel tips remain unknown.

Wind-Tunnel and Flow Conditions

These experiments were conducted in a continuous, freejet, blowdown, hypersonic wind tunnel. A conical nozzle with a 7.5° half-angle and an exit diameter of 4 in. was used. The stagnation-chamber conditions were $T = 2100 \pm 20^\circ\text{R}$ and $P = 1000 \pm 5.6$ psia. The resultant freestream Mach number was about 14.5 at the model tip and increased about 4% over the length of the longest model. The Mach number varied less than 1% over the maximum diameter of the models. Freestream conditions at the model tip are given in Table 1.

Surface Pressures

Pressure measurements were made on the cone and flare surfaces. Ten stainless-steel pressure lines ran about 0.030 in. below the model surface, at various positions about the model circumference (Fig. 2). Each line could therefore be used to determine pressures at many positions by changing the pressure-port locations. The models were aligned with the freestream using 3 or 4 pressure ports located on the flare in increments of 90° about the model circumference.

The primary sources of uncertainty in reported surface pressures are (in order of importance) freestream variations, flow misalignment, model distortions, and instrumental ac-

curacy. For all models except the 90° -flare model, the uncertainties are $p_w/p_\infty \pm 0.9$ for $\xi \leq \xi_0$ and $p_w/p_\infty \pm 1.0$ for $\xi_0 < \xi$. The uncertainties for the 90° -flare model are $p_w/p_\infty \pm 1.0$ for $\xi \leq \xi_0$, and $p_w/p_\infty \pm 1.5$ for $\xi_0 < \xi$. The scatter in the data is consistent with these uncertainties, except very near the outer edge of the flare, where the pressures were exceptionally sensitive to flow alignment. The lowest surface-pressure measurements were corrected by less than 4% for combined effects of thermal creep and lingering outgassing. The effects of orifice size on measured surface pressures were less than 1.0%. The fluid mechanical effects of orifice size were estimated from Talbot's¹⁸ supersonic empirical relation for cones. The influence of heat transfer on measured pressures, for small orifices and low densities, was determined from Potter, Kinslow, and Boylan's¹⁹ correction, using Horstman and Kussoy's²⁰ correlation of low-density heat-transfer data for cones.

Flow Surveys

Approximate velocity profiles, Mach number profiles and shock positions were obtained from Pitot surveys of separated and attached flows (Fig. 3). Initial Mach numbers were computed from measured wall pressures and Pitot impact pressures. Adiabatic isentropic relations were used for $M < 1$. The Rayleigh Pitot formula was applied when $1 \leq M$. Pressure uncertainties result in $M \pm 0.2$ for $M < 1$ and $M \pm 0.1$ for $1 \leq M$. Velocity points were then determined from the Mach numbers by assuming that the total temperature is given by Crocco's relation, $T_t = (T_{t,\infty} - T_w)(U/U_\infty) + T_w$. This assumption alone yields a maximum uncertainty of $U/U_\infty \pm 0.02$ roughly near the middle of the reported profiles. It becomes exact at the edge of the viscous layer and at the wall (when no slip). The Mach number directly at the edge of the viscous layer was obtained from an "experimental tangent-cone approximation," i.e., the inviscid cone solution defined by the measured pressure ratio p_w/p_∞ and $M_\infty = 14.4$. An error of $\pm 15\%$ in M_∞ would yield $U/U_\infty \pm 0.015$ for the reported profiles. The viscous-layer edge and the shock were located ($Y \pm 0.004$) using a transient technique¹⁷ that relied on variations in measurements of the rate of change of transducer pressures, as the probe traversed the flow. The high-velocity portions of these initial profiles are believed to be reliable.

The simple Pitot relations that were used to obtain the initial Mach number points become inaccurate at low Reynolds numbers and in rarefied flows (e.g., see Ref. 21). For

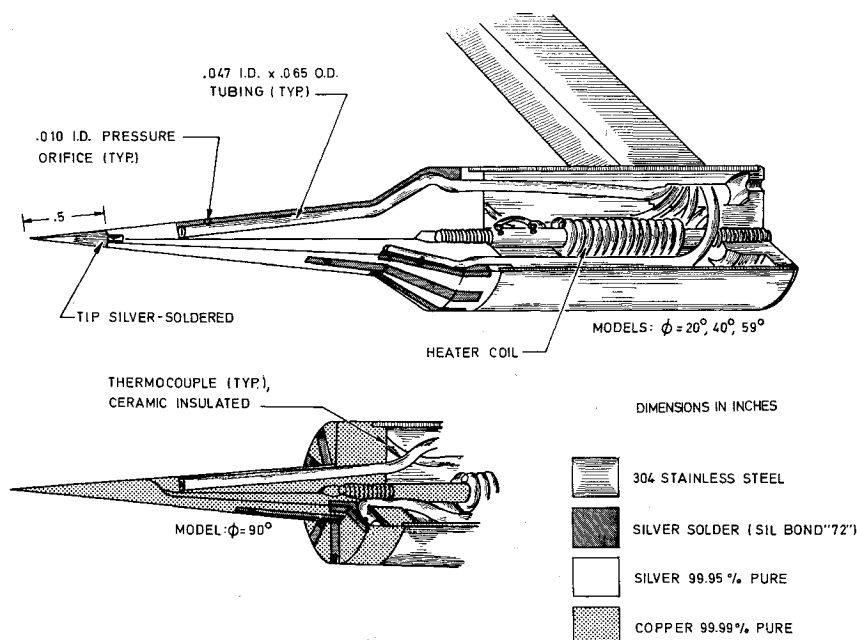


Fig. 2 Model construction.

Table 1 Freestream conditions

ϕ , deg	M_∞	L_∞ , in.	Re_∞ , in. ⁻¹	p_∞ , mmHg	T_∞ , °R	λ_∞ , in.
20°, 40°, 59°	14.45	27.51	2.9×10^4	0.0935	52.0	7×10^{-4}
90°	14.55	30.27	2.8×10^4	0.0893	51.7	7×10^{-4}
Uncertainty	$\pm 0.4\%$	$\pm 0.4\%$	$\pm 4.8\%$	$\pm 2.7\%$	$\pm 1.1\%$	

this reason, the reported initial velocity points become inconsistent with the wall conditions at low Reynolds numbers, about $Re \cdot d < 8$. This result qualitatively agrees with McMillan's calibration data for squeezed-tip probes at low velocities and low Reynolds numbers.²² Therefore, when velocity curves were faired, only points with $8 \leq Re \cdot d$ were taken into account. The velocity curves for attached flow were repeatedly faired to the wall by computing and correcting for slip velocity until $U_w \approx \lambda_w(dU/dY)_w$. The curves for separated flows were faired only once into the vicinity of the zero velocity point. Mach number curves, skin-friction coefficients, and displacement thicknesses were then computed from the faired velocity curves. These results should be viewed as experimental first approximations.

An inverse calculation and comparison may be carried out to partially assess the validity of the faired profiles. Ideal Pitot pressures, i.e., without viscous or rarefaction effects, and the local flow conditions can be computed from the faired profiles. The measured impact pressures (reported in Ref. 17) can then be referenced to these quantities and compared with probe calibration data. This was done¹⁷ in order to qualitatively compare such results, for the current probes, with published data for a variety of other probes ($0.1 \leq M \leq 18.3$, $2 \leq Re \cdot d \leq 10^3$). The Pitot pressure corrections implied by 9 profiles reported here 1) are qualitatively consistent, 2) display the proper form, and 3) are large enough to demonstrate that the differences between the initial velocity points and the faired velocity curves are due to viscous and rarefaction effects on the measured impact pressures.

The influence of the probe on the flow was checked during the surveys by measuring changes in surface pressures for 14 representative probe positions. The probe did not alter the sampled pressures by more than a few percent beyond the experimental uncertainty in the surface pressure data, except in one vicinity. At points near the flare edge, directly in the wake of the probe, pressures increased by as much as 40%. However, the surface pressures in this particular region were extremely sensitive to local flow angularity, as was determined by pitching the models. This limited study therefore implied that the probes did somewhat influence the flow upstream of the probe, but not enough to alter the level of approximation of the results.

Flow Upstream of Separation Interaction

The first task is to define the rather complex flow that existed just upstream of the beginning of separation interaction. Tempilag[†] fluid flow, carbon tufts, and the Pitot

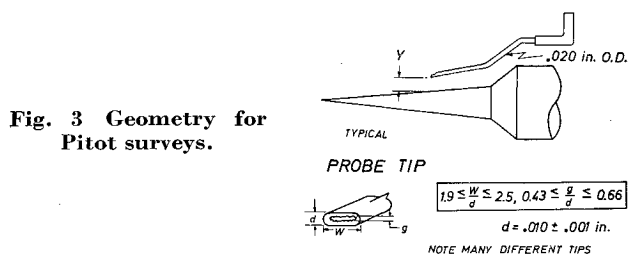


Fig. 3 Geometry for Pitot surveys.

[†] Tempilag is a brand name of paints that indicate surface temperatures by melting ($500^\circ\text{R} \leq T_w \leq 2900^\circ\text{R}$). In the fluid form these paints become flow indicators, even at very high temperatures.

probe surveys indicated that the 40°, 59°, and 90° flares induced significant separation regions, whereas the 20° flare did not. All pertinent available data indicate that separation interaction begins at the point where the surface pressures first begin to sharply increase (Fig. 4). Table 2 lists approximate flow conditions at ξ_0 . The flow conditions at the edge of the viscous layer were estimated using the experimental tangent-cone approximation described in the previous section.

Tip-Bluntness Effects

Tip-bluntness effects on the surface pressures carried more than 1000 tip diameters downstream, despite low Reynolds numbers ($20 \leq Re \cdot d \leq 81$) and sizeable Knudsen numbers ($0.4 \leq \lambda_\infty/t \leq 1.0$). The surface pressures are plotted in an expanded scale in Fig. 5. The scatter in the data for each model is within the experimental uncertainty. However, the differences among measurements for the various models definitely cannot be explained by experimental uncertainties. The influence of tip bluntness is brought out by replotting the distance coordinate in terms of tip diameters (Fig. 6). The data for the 90°-flare model could have been influenced by differences in freestream conditions and tip temperatures. Thus, the flows just prior to separation interaction were somehow influenced by the minutely blunted tips.

A comparison of inviscid blunt-cone solutions^{23,24} and viscous sharp-cone theory²⁵ implies that viscous-interaction effects should have been completely dominant in the region of the current measurements. This suggests that minute tip bluntness may have indirectly influenced the flow in this region by altering the early history of the viscous layer near the tip. For thick viscous layers, such effects may carry further downstream that would otherwise be expected.

Effects of Viscous Interactions and Freestream Nonuniformities

Viscous interaction parameters for defining the boundary between the strong- and weak-interaction regimes for cones in hypersonic flow have not been firmly established. None-

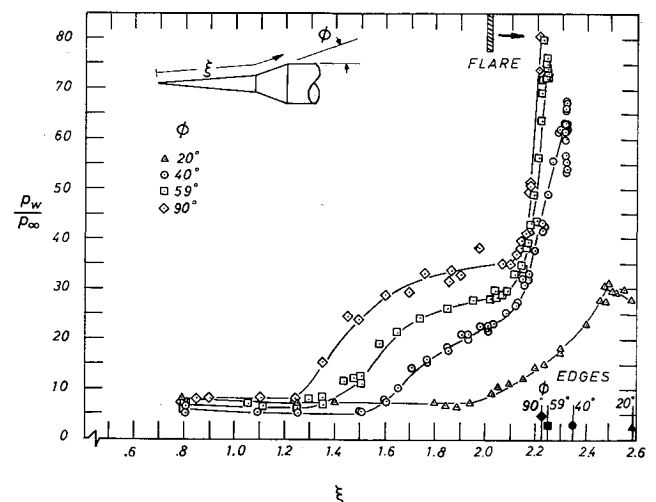


Fig. 4 Surface pressures.

Table 2 Conditions at beginning of separation interaction

ϕ , deg	ξ_0 , in.	M_0	Re_{ξ_0}	δ_0^* , in.	C_{f_0}	$p_{w,0}/p_\infty$	$U_{w,0}/U_\delta$, %	$\mu_{w,0}$ lbf sec/ft ²	T_w/T_0
40	1.48	10.8	8.5×10^4	0.100	0.385×10^{-2}	5.02	12.8	7.7×10^{-7}	16.2
59	1.33	10.2	6.7×10^4	0.092	0.436×10^{-2}	6.40	12.9	7.7×10^{-7}	14.4
90	1.24	9.75	5.8×10^4	0.087	0.472×10^{-2}	7.33	13.4	7.7×10^{-7}	13.3

theless, it is clear that the surface-pressure measurements upstream of ξ_0 (Fig. 5) are in the weak-interaction regime that is defined^{25,26} by the basic criteria $\tan^{-1}(d\delta^*/dx) < \theta_c$. Using the experimental tangent-cone approximation, one may show that $1.33 < \theta_c/\tan^{-1}(d\delta^*/dx) \leq 2.85$ for these measurements.

Figure 7 shows the predictions of Hoelmer, and Saarlus²⁵ simplification of Talbot, Kuga, and Sherman's²⁷ sharp-cone weak-interaction theory. The theory is corrected to non-uniform freestream conditions in three ways. The first is Hall's²⁸ inviscid sharp-cone correction for a slightly non-uniform hypersonic freestream, i.e.,

$$p_w = P_w \{1 - 4x/L_\infty + \dots 0[(x/L_\infty)^2]\} \quad (1)$$

The second is a first-order viscous-flow correction for a slender cone. This relation may be obtained from Newtonian-impact relations and Hall's hypersonic source-flow relations by neglecting differences in viscous-layer growth for the two cases¹⁷ and dropping terms $0[(x/L_\infty)^2]$

$$p_w = P_w \{1 - 2[x + (y_w + \delta^*)/\tan\alpha_b]/L_\infty\} \quad (2)$$

The third is an empirical so-called "buoyancy-correction" that has often been used,

$$p_w = P_w + p_a - p_\infty \quad (3)$$

The magnitudes of the corrections (Fig. 7) indicate that the slightly nonuniform freestream had a considerable influence on the surface pressures upstream of ξ_0 . Pressure data for the sharpest model tip are compared, in Fig. 7, with the corrected theoretical values. The comparison implies that viscous effects were dominant, although tip bluntness influenced some, if not all, of the data.

Skin-Friction Coefficients and Displacement Thicknesses

Skin-friction coefficients were obtained from two experimental velocity profiles in flows uninfluenced by separation. The values were $C_f = 0.40 \times 10^{-2}$ (Fig. 8) and $C_f = 0.46 \times 10^{-2}$ (Fig. 9, $\xi = 1.21$). These values are only about 1% above those predicted when Raat's²⁹ constant-pressure theory is applied as an experimental locally similar solution. The skin-friction coefficients are computed from Raat's theory by taking the local, experimental, tangent-cone flow-conditions as the flow conditions at the edge of the viscous layer. This

method of applying Raat's theory was used to obtain C_{f_0} (Table 2). Displacement thicknesses from the two profiles are $\delta^* = 0.099$ (Fig. 8) and $\delta^* = 0.086$ (Fig. 9, $\xi = 1.21$). A linear interpolation between these two values was used to estimate δ_0^* where necessary (Table 2).

Rarefaction

Slip velocities at ξ_0 were at least 10% of the external velocity. The slip velocities given by two velocity profiles are $U_w/U_\delta \approx 0.1$ (Fig. 8, $\xi = 1.48$; and Fig. 9) when λ_w is obtained from Probstein's³⁰ relation, as calculated in Ref. 31. However, the related slip velocities are about 30% higher if λ_w is calculated from³² $\lambda_w = 1.26 \mu_w (RT_w)^{1/2}/p_w$. Slip velocities at ξ_0 were estimated using this last expression³² for $\lambda_{w,0}$ and the C_{f_0} values. They are related by $U_{w,0} \approx \lambda_{w,0} C_{f_0}/\mu_w$. The reported slip velocities (Table 2) are higher than one might have expected from available flat-plate data,³³ although these data do not extend to such low values of the rarefaction parameter ($M_\infty [C_\infty/Re_\infty \xi]^{1/2} \approx 0.06$). McCroskey, Bogdonoff, and Genchi³⁴ have shown that cone flows can be considerably more rarefied than flows over flat plates, even for the same freestream conditions. The characteristically higher skin-friction coefficients for a cone should generally result in proportionally greater slip velocities, when the flow conditions at the edges of the viscous layers are equal and the wall temperatures are the same for both cases.¹⁷

Separation Interaction

Velocity Profiles and Shock Shapes

Figures 9 and 10 show velocity profiles in regions of separation interaction. The velocities are shown along the x axes, but the flow angles are unknown. The flow was considered reversed when the Pitot impact pressures were less than or equal to the wall pressures. These profiles bracket the separation points, which must of course lie between the attached and the separated profiles. The viscous layers and shocks (Fig. 10) run close to each other and have smooth, low-sloping, concave shapes. The shocks must have been modified by a series of light compression waves originating from the viscous layers.

Surface Pressures and Laminar Separation

The surface-pressure distributions for the separated flows each have two inflection points upstream of the reattach-

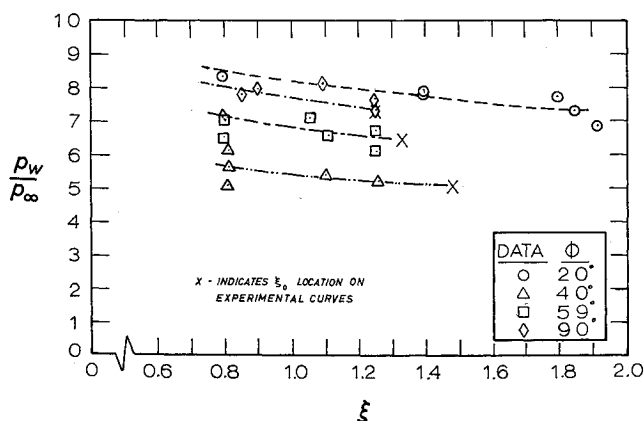


Fig. 5 Cone surface pressures uninfluenced by separation.

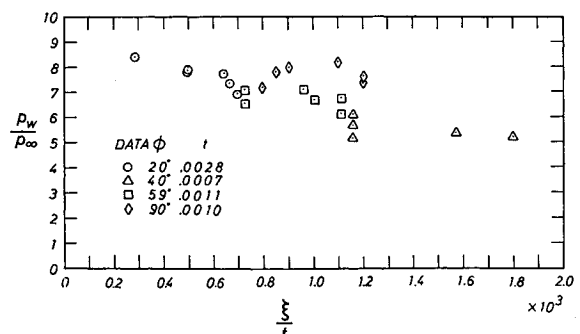


Fig. 6 Influence of tip bluntness.

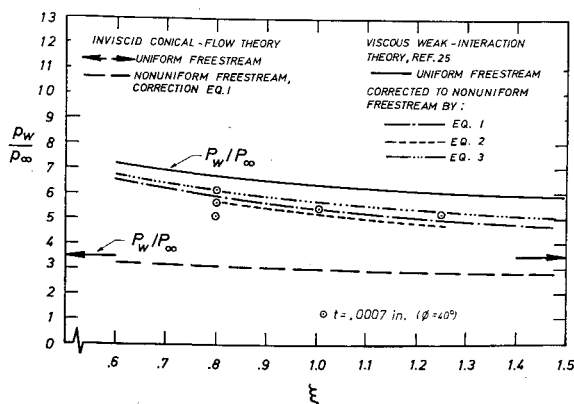


Fig. 7 Weak-interaction theory corrected to nonuniform freestream conditions.

ment pressure rise (Fig. 4: $\phi = 40^\circ, 59^\circ$, and 90°). A third inflection point should occur during the reattachment pressure rise, because the surface pressures decrease near the outer edge of the flare, as the flow turns the corner. These results are, therefore, consistent with Kuehn's² criteria for identifying incipient separation from the appearance of three such inflection points. Figure 4 shows that only the 90° flare produced a pressure plateau ($dp_w/dx = 0$) at the second inflection point. Previously expressed views^{35,36} that pressure plateaus are characteristic of laminar compression-separation regions are not supported by these data or by Kuehn's² results. On the other hand, pressure plateaus certainly can arise and have often been reported^{2,37-39} for such cases (doubts in Ref. 40). The development of the plateau is further discussed in the next section.

In order to show that the current separation regions were laminar, conditions at the viscous-layer edge just prior to the reattachment pressure rise are compared with transition data^{8,37,41} for a number of bodies (Fig. 11). The flow properties, M_δ and Re_{pr} , at the start of the reattachment pressure rise were determined using the experimental tangent-cone approximation. Figure 11 indicates that the current separation regions were well within the pure laminar domain.

Free Interaction

Free interaction is said to exist when separation interaction is governed by a local interaction between the viscous layer and the inviscid external flow.^{37,42} The cause of separation (ramp, flare, etc.) poses the reattachment conditions that establish where separation interaction will begin. From that point on, the separation-interaction process is completely independent of the downstream reattachment conditions. Semiempirical surface-pressure correlations provide

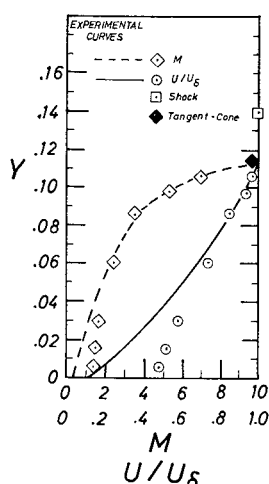


Fig. 8 Survey in flow undisturbed by separation, $\phi = 40^\circ$.

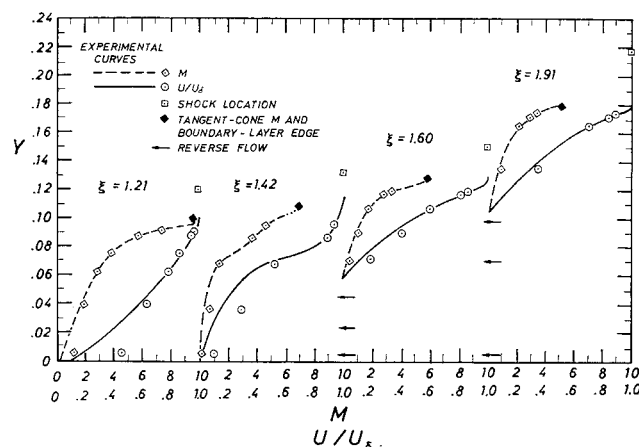


Fig. 9 Flow surveys within and just upstream of separation-interaction region, $\phi = 90^\circ$.

the major source of prior evidence that free interaction exists for many cases of laminar or slightly transitional, two-dimensional, compression-separation regions. Most of these correlations are based on Chapman, Kuehn, and Larson's³⁷ supersonic free-interaction analysis and extensions^{35,42} of this analysis. The extensions^{35,42} also apply only for supersonic separation, since the linearized supersonic pressure relation is retained. To the author's knowledge, only one prior correlation of this kind has been obtained for axisymmetric flow with significant transverse curvature. De Roquefort¹⁴ transformed Erdos and Pallone's³⁵ flat-plate relations and, with the aid of the tangent-cone approximation, correlated separation-interaction pressures on a cone in supersonic flow. De Roquefort's results,¹⁴ Ferguson and Schaefer's⁴ supersonic pressure correlation, and Wood's¹⁰ correlations of plateau angles in hypersonic flow seem to be the only prior evidence of free interaction for axisymmetric compression-separation regions.

Lewis, Kubota, and Lees⁴² clarified Chapman, Kuehn, and Larson's³² analysis on the basis of similarity. Similarity is assured if all governing flow relations and conditions can be nondimensionalized by a single set of variables. But in a Chapman analysis, only the Navier-Stokes relations, evaluated at the wall, and an expression for the pressure at the edge of the viscous layer are used. As a final step, it is therefore necessary to justify the results by more rigorous means. Experimental results^{35,37,42} and a more rigorous theoretical analysis⁴³ support prior Chapman analyses.

A simple Chapman analysis is also employed here to obtain similarity variables for hypersonic, axisymmetric, laminar, separation interaction on a slender cone ($M_\infty \alpha_b \gg 1$, $\theta_c \ll 1$). The axisymmetric Navier-Stokes equations are evaluated at the wall. A term τ_w/y_w , which should appear on the right side of Eq. (4) is neglected yielding

$$dp_w/dx \approx (\partial \tau / \partial y)_w \quad (4)$$

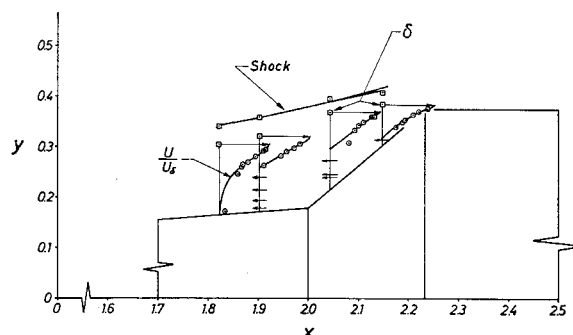


Fig. 10 Flow surveys within separation-interaction and reattachment regions, $\phi = 40^\circ$.

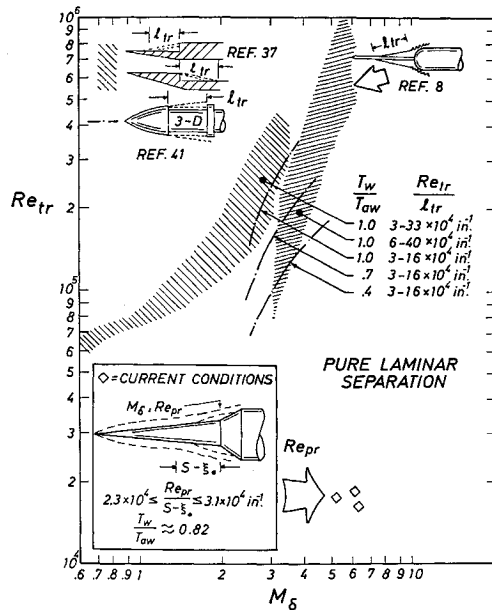


Fig. 11 Comparison of current flow conditions with transition data.

A local Newtonian-pressure approximation, based on the conditions at ξ_0 , provides the pressure at the edge of the boundary layer. It is assumed that $p_w = p_\delta$. Possible effects of upstream viscous interactions and freestream non-uniformities on the inviscid pressures are neglected. Thus,

$$Cp_0 \approx 2(d\delta^*/dx)^2 \quad (5)$$

Since the conservation of energy is not considered, significant differences in heat transfer are not allowed.

It is hypothesized that free interaction exists and that the following variables are the basic similarity variables for this case:

$$\begin{aligned} \bar{X} &= (x - x_0)/l_i, & \bar{\delta}^* &= \delta^*/\delta_0^* \\ \bar{Y} &= Y/\delta_0^*, & \bar{\tau}_w &= \tau_w/\tau_{w0} \end{aligned} \quad (6)$$

These variables are the same as those employed by Lewis, Kubota and Lees,⁴² except for the use of $\bar{\delta}^*$ instead of Θ . If the current basic variables [Eq. (6)] are used for supersonic flow, then the results will be exactly the same as those given in Ref. 42 (and Ref. 35), providing $(d\delta^*/dx)_0$ is not included in the linearized supersonic pressure relation. Now for hypersonic flow, we have two equations [Eqs. (4) and (5)] and two remaining unknowns ($\bar{P}$ and l_i). When one solves for $\bar{P}$ and l_i so as to leave the transformed equations non-dimensional, the following result is obtained:

$$\bar{P} = Cp_0/C_{f0}^{2/3} \quad (7)$$

$$\bar{X} = C_{f0}^{1/3}(x - x_0)/\delta_0^* \quad (8)$$

First-order transverse curvature influences the free-interaction similarity variables [Eqs. (6-8)] only through the magnitudes of the initial skin-friction coefficients and displacement thicknesses. These variables should, therefore, also yield approximate correlation variables for two-dimensional, hypersonic, laminar, separation-interaction regions.

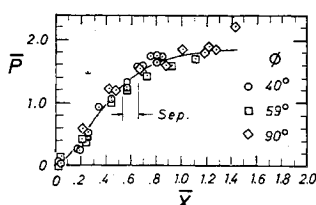


Fig. 12 Correlation of surface pressures in separation-interaction region.

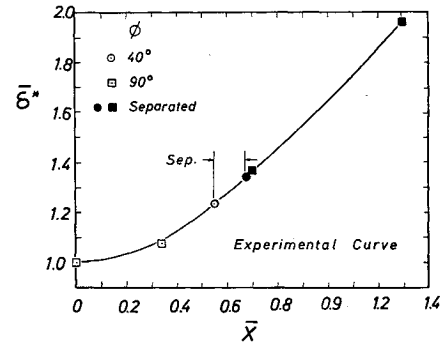


Fig. 13 Correlation of displacement thicknesses in separation-interaction region.

The pressure and displacement thickness correlations presented in Figs. 12 and 13 imply that free interaction existed for these separation regions. The separation-point brackets from the Pitot surveys are also shown. Note that the surface-pressure distributions that did not have a plateau ($\phi = 40^\circ$ and 59° , Fig. 4) climb partially up the correlation curve (Fig. 12). Similar results were obtained by Lewis, Kubota, and Lees⁴² who correlated surface pressures for supersonic separation-interaction regions on flat plates. The pressure plateaus, developed as the size of the separation regions, increased. The surface pressures for separation-interaction regions, with and without plateaus, were also shown to have a single functional dependence upon the flow conditions at the beginning of separation interaction. Thus, the flow in these regions cannot be directly dependent on the reattachment conditions or the downstream distance to the beginning of the reattachment pressure rise. This is free interaction.

The experimental displacement thicknesses (Fig. 13) were computed assuming zero velocity in reverse-flow regions. The resultant error is believed to be small because the densities in these regions were very low, due to the high wall temperatures. In any case, the associated error in δ^* can not be more than 7%, because this is the maximum difference between the displacement thicknesses reported and the edges of the separated viscous layers. Since the reverse flows would increase the displacement thicknesses, the reported values are inner limits and the edges of the viscous layer are outer limits.

Influence of Freestream Nonuniformity

Equations (1) and (2) indicate changes in surface pressures, due to freestream nonuniformity, of up to 30% for these separation-interaction regions. This qualitatively indicates that such an effect was significant. However, the exact influence of the nonuniformity is unknown because the derivations^{17,28} of Eqs. (1) and (2) do not allow for adjustments in the separated flow and associated surface-pressure changes. The separation-interaction process itself could not have been substantially altered, because both surface pressures and displacement thicknesses (Figs. 12 and 13) are correlated by the free-interaction variables $\bar{P}$, $\bar{\delta}^*$, and $\bar{X}$. This suggests that the freestream nonuniformity influenced the flow in this region, mainly by shifting the beginning of separation interaction and altering the flow characteristics at that point.

Summary and Conclusions

This has been an experimental investigation of laminar separation interaction on a 5° half-angle cone in a slightly nonuniform $M_\infty \approx 14.5$ flow. The viscous layers at the beginning of separation interaction were about as thick as the body radius, were in the weak-interaction regime, and had about 10% velocity slip at the wall ($U_w/U_\delta \approx 0.1$). The upstream flows were substantially influenced by minute tip-bluntness and freestream nonuniformity. The smallest separation region did not have a pressure plateau, but one

gradually developed as the size of the separation region was increased by raising the flare angle from 40° to 90° . The influence of freestream nonuniformity on the separated flow was not negligible. Nevertheless, three considerably different separation-interaction surface-pressure distributions were correlated by hypersonic variables obtained from a Chapman analysis. Since these variables were based only on flow conditions at the beginning of separation interaction, this is evidence of free interaction.

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